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# Fusion of individual preference orderings in an ordinal semi-democratic decision-making framework

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## Extended abstract

A general problem, which may concern practical contexts of different nature, is to aggregate multi-agent orderings of different alternatives into a single "consensus fused ordering". It is assumed that there are $M$ decision-making agents[*] $D_1, D_2, \ldots, D_M$, each of which defines an ordering of $n$ alternatives $a$, $b$, $c$, etc.. For any two alternatives $a$ and $b$, this ordering allows statements like $a > b$, $a \sim b$, $b > a$, where symbols ">" and "~" respectively mean "strictly preferred to" and "indifferent to". Also, it is assumed a rank-ordering over the agents, based on their individual importance. This other ordering admits relations of strict preference and indifference too.

For the purpose of example, Fig.1 shows the preference orderings by four fictitious agents ($D_1$ to $D_4$,). In this case, the agents' importance ordering is assumed to be $D_1 > (D_2 \sim D_3) > D_4$.

The problem of interest is fairly general [1, 2, 3, 4] and can be applied to a variety of practical contexts, such as:

[*] By a decision-making agent we will consider any of a wide variety of different types of entities. Examples could be human beings, individual criteria in a multi-criteria decision process or software based intelligent agents on the Internet.

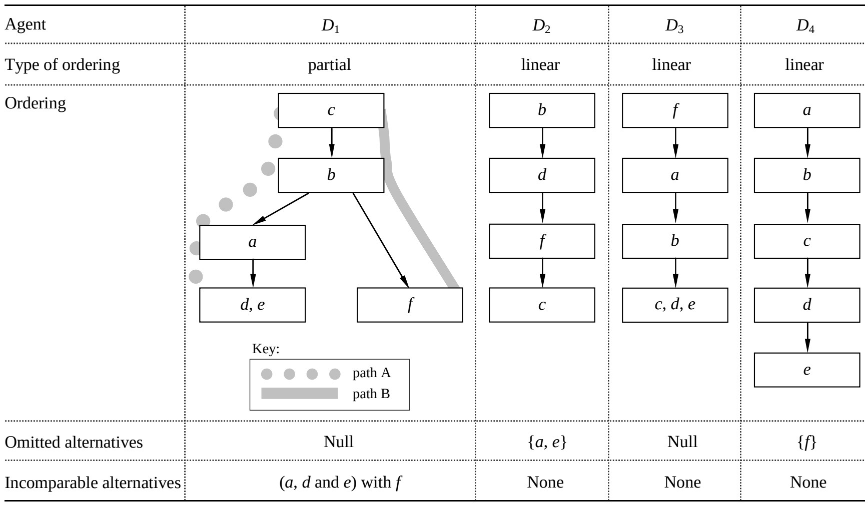


Fig. 1. Graphical representation of the preference orderings by four fictitious agents. The alternatives of interest are *a*, *b*, *c*, *d*, *e* and *f*. The agents' importance ordering is $D_1 > (D_2 \sim D_3) > D_4$. The preference ordering by $D_1$ is *partial* (with some incomparable alternatives) while the remaining are *linear* (in some cases with omitted alternatives).

- multi-criteria decision processes; e.g., determination of the best location where to install a new manufacturing plant, based on several criteria – such as road/railway infrastructure, electrical supply, presence of satellite industries, labour cost, etc. – where criteria are just ordered, with no precise weight;
- intelligent customization of data displayed on Internet sites, based on several ordered criteria (e.g., for advertising decisions);
- synthesis of customer requirements, evaluated on ordinal scales by a sample of questionnaire/interview respondents, in the Quality Engineering field.

An interesting aspect of this problem is the importance hierarchy of agents, which is simply given by an *ordering* and not defined on an *interval* or *ratio* scale [5]. Contrary to many other decision-making problems, in this case a weight depicting the absolute importance of each agent is not defined. For this reason, this decision-making framework can be denominated as "ordinal semi-democratic". The adjective "semi-democratic" indicates that agents do not necessarily have the same importance, while "ordinal" indicates that their hierarchy is defined by a crude ordering. This makes the set of the possible solutions to the problem relatively wide, since they may range between the two extremes of (i) *full dictatorship* – in which the fused ordering coincides with the preference ordering by the most important agent (dictator) – and (ii) *full democracy*, where all agents are considered as equi-important.

Over ten years ago, Yager [6] proposed an algorithm to address the above problem in a relatively simple, fast and automatable way. Unfortunately, this algorithm has two important limitations: (i) the resulting fused ordering may sometimes not reflect the preference orderings by the majority of agents [7] and (ii) it is only applicable to (non-strict) *linear* orderings without incomparabilities and omissions of the alternatives of interest (such as that one by $D_3$ in Fig. 1).

The objective of this paper is to enhance the algorithm by Yager so as to overcome its limitations and adapt to less stringent preference orderings. The new algorithm can be interpreted as a generalization of that by Yager and has two main advantages: (i) it better reflects the individual multi-agent preference orderings and (ii) it is more flexible, since it is applicable to (non-strict) *partial* orderings (such as that one exemplified in Fig. 1), which admit omitted or incomparable alternatives. Also, it is automatable, continues to satisfy the properties of the standard Yager's algorithm, and can be applied to a larger variety of practical contexts, providing more realistic results. Tab. 1 summarizes the requirements that the new algorithm is supposed to meet.

Tab. 1. Requirements of the new algorithm. Symbols "✓" and "×" identify those satisfied and non-satisfied by the standard Yager's algorithm.

| Requirements | Yager's |
|---|---|
| 1. Each agent can have its own individual preference ordering over the alternatives. | ✓ |
| 2. Agents (can) have a hierarchical importance ordering. | ✓ |
| 3. The algorithm should be automatable. | ✓ |
| 4. The preference orderings should include the possibility of ties between two or more alternatives. | ✓ |
| 5. The preference orderings should include the possibility of omitting one or more alternatives. | × |
| 6. The preference orderings should include the possibility of incomparability between two or more alternatives. | × |
| 7. The logic for selecting alternatives should reflect the preference ordering for the majority of agents. | × |
| 8. All agents (can) have the same importance (*full democracy*). | × |

A potential limitation of the new algorithm is related to the mechanism for aggregating and/or comparing elements from different preference orderings. The underlying assumption is that the degree of preference of the alternatives in different preference orderings depends on their relative position.

The description of the new algorithm and its potential is supported by several practical examples.

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